

THE INVERSION FORMULAS FOR SOME TWO-DIMENSIONAL
 VOLTERRA INTEGRAL EQUATIONS OF THE FIRST KIND

V.F. Volkodavov and I.N. Rodionova

Various boundary value problems for some hyperbolic equations of the third order in the three-dimensional Euclidean space are reduced to two-dimensional Volterra integral equations of the first kind, which have a unique solution. Thus, let us consider, for instance, the equation

$$L(u) = (z + y - x)U_{xyz} - \beta U_{xy} + \alpha U_{yz} = 0, \quad 0 < \alpha, \beta < 1, \quad (1)$$

in a domain Ω — a pyramid bounded by the planes $x = h, y = 0, z = 0, z = x - y$. In Ω we shall state the following boundary value problems for equation (1).

Find in Ω a solution of (1), which will satisfy the boundary conditions:

problem I.

$$U(x, y, 0) = \psi(x, y), \quad (2)$$

$$U(x, y, x - y) = \tau(x, y), \quad 0 \leq y \leq x, \quad 0 \leq x \leq h, \quad (3)$$

$$\lim_{z \rightarrow (x-y)-0} \frac{\partial U}{\partial z} = \nu(x, y); \quad (4)$$

problem II.

$$U(h, y, z) = \Phi(y, z), \quad 0 \leq y \leq h - z, \quad 0 \leq z \leq h, \quad (5)$$

as well as (3) and (4);

problem III.

$$U(x, 0, z) = \chi(x, z), \quad 0 \leq x \leq z, \quad 0 \leq z \leq h$$

and conditions (3), (5).

Problems I–III can be reduced, respectively, to the following integral equations:

$$\int_0^{x-y} \int_y^{x-z} N_1(s, \xi)(x - \xi - s)^\alpha (\xi - y)^\beta d\xi ds = f_1(x, y), \quad (6)$$

$$\int_z^{h-y} \int_0^y N_2(s, \xi)(h - \xi - s)^\alpha (y - \xi)^\beta d\xi ds = f_2(y, z), \quad (7)$$

$$\int_z^{h-y} \int_y^{h-s} N_3(s, \xi)(h - \xi - s)^\alpha (\xi - y)^\beta d\xi ds = f_3(y, z). \quad (8)$$

Set

$$H_1 = \left\{ (x, y) \mid \begin{array}{l} 0 < y < x \\ 0 < x < h \end{array} \right\}, \quad H_2 = \left\{ (y, z) \mid \begin{array}{l} 0 < y < h - z \\ 0 < z < h \end{array} \right\}.$$

Assume that the right sides in (6)–(8) satisfy the following

Conditions A. $f_1(x, y) = (x - y)^{3+\beta+\varepsilon} \varphi(x, y), \varphi_{xy}^{(IV)} \in C(\overline{H_1})$.

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