Russian Mathematics (Iz. VUZ) *Izvestiya VUZ. Matematika*
Vol. 42, No. 9, pp.28–30, 1998 *UDC 517.968*

THE INVERSION FORMULAS FOR SOME TWO-DIMENSIONAL VOLTERRA INTEGRAL EQUATIONS OF THE FIRST KIND

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Various boundary value problems for some hyperbolic equations of the third order in the three-dimensional Euclidean space are reduced to two-dimensional Volterra integral equations of the first kind, which have a unique solution. Thus, let us consider, for instance, the equation

$$L(u) = (z + y - x)U_{xyz} - \beta U_{xy} + \alpha U_{yz} = 0, \quad 0 < \alpha,\ \beta < 1, \tag{1}$$

in a domain Ω — a pyramid bounded by the planes $x = h$, $y = 0$, $z = 0$, $z = x - y$. In Ω we shall state the following boundary value problems for equation (1).

Find in Ω a solution of (1), which will satisfy the boundary conditions:
problem I.

$$U(x, y, 0) = \psi(x, y), \tag{2}$$

$$U(x, y, x - y) = \tau(x, y), \quad 0 \le y \le x, \ \ 0 \le x \le h, \tag{3}$$

$$\lim_{z \to (x-y)-0} \frac{\partial U}{\partial z} = \nu(x, y); \tag{4}$$

problem II.

$$U(h, y, z) = \Phi(y, z), \quad 0 \le y \le h - z, \ \ 0 \le z \le h, \tag{5}$$

as well as (3) and (4);
problem III.

$$U(x, 0, z) = \chi(x, z), \quad 0 \le x \le z, \ \ 0 \le z \le h$$

and conditions (3), (5).

Problems I–III can be reduced, respectively, to the following integral equations:

$$\int_0^{x-y} \int_y^{x-z} N_1(s, \xi)(x - \xi - s)^\alpha (\xi - y)^\beta d\xi\, ds = f_1(x, y), \tag{6}$$

$$\int_z^{h-y} \int_0^{y} N_2(s, \xi)(h - \xi - s)^\alpha (y - \xi)^\beta d\xi\, ds = f_2(y, z), \tag{7}$$

$$\int_z^{h-y} \int_y^{h-s} N_3(s, \xi)(h - \xi - s)^\alpha (\xi - y)^\beta d\xi\, ds = f_3(y, z). \tag{8}$$

Set

$$H_1 = \left\{ (x, y) \,\middle|\, \begin{array}{l} 0 < y < x \\ 0 < x < h \end{array} \right\}, \qquad H_2 = \left\{ (y, z) \,\middle|\, \begin{array}{l} 0 < y < h - z \\ 0 < z < h \end{array} \right\}.$$

Assume that the right sides in (6)–(8) satisfy the following
Conditions A. $f_1(x, y) = (x - y)^{3+\beta+\varepsilon}\varphi(x, y)$, $\varphi^{(\mathrm{IV})}_{xxyy} \in C(\overline{H}_1)$.