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CONSTRUCTION OF PARAMETERIZED SOLUTIONS OF LINEAR OPERATOR EQUATIONS ON THE BASE OF A MODIFIED GALYORKIN METHOD

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1. Introduction. In solving a wide class of applied problems the necessity arises to construct so-called parameterized (known up to parameters) analytic solutions, which obey with an admissible exactness the operator equations under consideration for given domains of possible values of characteristic parameters of the problems to be investigated.

Below we shall expose the unified approach to the construction of parameterized analytic solutions of linear operator equations (algebraic, differential, integral, etc.). In the base of this approach the continuous dependence of solution on the parameters is put as well as the idea to use support solutions, which was first published in [1], [2], and [3]. The development of the theory is carried out in a new projective-parametric statement which generalizes the theory of projective methods of the Galyorkin type (see [4]–[7]) to the case considered by the author.

2. Projective-parametric statement of the problem. Assume that two normed spaces X and Y are given. In each of them a complete subspace $\widetilde{X} \subset X$ or $\widetilde{Y} \subset Y$, respectively, is selected. We shall assume that a continuous linear operator $\widetilde{\Phi}(\omega)$ is given which projects Y to $\widetilde{Y}$, and also a linear operator equation (which is termed exact, by analogy with [4])

$$K_1(\omega)x(\omega) \equiv G(\omega)x(\omega) - \lambda T(\omega)x(\omega) = y(\omega), \quad x \in X, \ \ y \in Y, \tag{1}$$

where $G(\omega)$ and $T(\omega)$ are continuous linear parameterized operators which send X to Y and depend on a real vector parameter $\omega \in \Omega \subset R^M$ (Ω is an open bounded connected domain, $M \in \{1, 2, \dots\}$), λ being a constant.

In addition, we assume that with respect to the operator $G(\omega)$ for all $\omega \in \Omega$ the following is valid

1) the operator $G(\omega)$ has a continuous inverse operator;
2) the operator $G(\omega)$ establishes one-to-one correspondence between $\widetilde{X}$ and $\widetilde{Y}$, i. e., $G(\omega)\widetilde{X} = \widetilde{Y}$, and therefore $G^{-1}(\omega)\widetilde{Y} = \widetilde{X}$.

Along with (1) we shall consider the approximate equation

$$\widetilde{K_1}(\omega)\widetilde{x}(\omega) \equiv G(\omega)\widetilde{x}(\omega) - \lambda\widetilde{T}(\omega)\widetilde{x}(\omega) = \widetilde{\Phi}(\omega)y(\omega), \tag{2}$$

where $\widetilde{T}(\omega)$ is a continuous linear parameterized operator acting from $\widetilde{X}$ to $\widetilde{Y}$.

For all $\omega \in \Omega$, we introduce the following conditions:

1) for any $\widetilde{x}(\omega) \in \widetilde{X}$ the following inequality holds

$$\|\widetilde{\Phi}(\omega)T(\omega)\widetilde{x}(\omega) - \widetilde{T}(\omega)\widetilde{x}(\omega)\| \le \mu_0\|\widetilde{x}(\omega)\|, \tag{3}$$