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OPTIMAL ALGORITHMS OF RECONSTRUCTION OF FUNCTIONS AND COMPUTATION OF INTEGRALS ON A CLASS OF INFINITELY DIFFERENTIABLE FUNCTIONS

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Introduction

In [1] a series of important problems of Computational Mathematics were formulated, among them the problems of computation of diameters and construction of optimal cubature formulas on various classes of functions. In particular, the problems of computation of diameters on the classes $Q_r(\Omega, M)$, $H_p(\{m_j\}, A)$ and construction of optimal cubature formulas on the class of functions $H_p(\{m_j\}, A)$ were posed. The problem of computation of diameters on the class of functions $Q_r(\Omega, M)$ was solved in [2] (see also [3]). The optimal cubature formulas on the class of functions $Q_r(\Omega, M)$ and some generalizations of the class were constructed by the author in [4] (see also [3]).

In the present article we shall give a solution of the problem of diameters' computation and optimal cubature formulas' construction on the class of infinitely differentiable functions $H_p(\{m_j\}, A)$. Since in the constructions below we shall essentially use the quantities m_n, therefore it is natural to select a subclass of functions for which our arguments will be carried out. Practically, an expansion of the results obtained to other subclasses can be made almost literally.

1. Auxiliary propositions

Definition 1.1. Let $\Omega = [-1,1]^l$, $l = 1,2,\dots$, $r = 1,2,\dots$, $0 \le \gamma < 1$. A function $f(x_1,\dots,x_l)$ belongs to the class $B_{r,\gamma}(\Omega)$ if the following conditions are fulfilled

$$\max_{x\in\Omega} |\partial^{|v|} f(x_1,\dots,x_l)/\partial {x_1}^{v_1} \cdots \partial x_l^{v_l}| \le A^{|v|}|v|^{|v|}$$

for $0 \le |v| \le r$,

$$|\partial^{|v|} f(x_1,\dots,x_l)/\partial x_1^{v_1} \cdots \partial x_l^{v_l}| \le A^{|v|}|v|^{|v|}/(\rho(x,\Gamma))^{|v|-r-1+\gamma}$$

for $r < |v| < \infty$.

Here $x = (x_1,\dots,x_l)$, $|v| = v_1 + \dots + v_l, \rho(x,\Gamma)$ is the distance between the point x and the boundary Γ of the domain Ω, which can be computed via the formula

$$\rho(x,\Gamma) = \min_{1\le i\le l} \min(|-1-x_i|, |1-x_i|),$$

the constant A does not depend on $|v|$.

Let us recall the definitions of the Babenko and Kolmogorov diameters.

Let B be a Banach space, $\Pi : X \to \overline{X}$ be a mapping of a compact $X \subset B$ onto a finite-dimensional space $\overline{X}$.

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