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# UNSTABLE AND MULTIPLE ELEMENTS OF THE SPECTRUM IN A SYSTEM OF SINGULARLY PERTURBED DIFFERENTIAL EQUATIONS

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## Introduction

We consider the problem

$$
\begin{aligned}
&L_\varepsilon W(x,\varepsilon) \equiv \varepsilon^2 W'(x,\varepsilon) - A(x)W(x,\varepsilon) = h(x),\\
&BW(0,\varepsilon) + CW(x_0,\varepsilon) + DW(a,\varepsilon) = \varepsilon^{-1}\alpha + W^0 \text{ for } \varepsilon \to +0, \quad x \in I = [0; a].
\end{aligned} \tag{1}
$$

Here $A(x)$ is a square matrix of $n$-th order, $h(x)$ is a known vector-function, $W(x,\varepsilon)$ is the desired vector-function, $B = \{b_{ij}\}$, $C = \{c_{ij}\}$, and $D = \{d_{ij}\}$ are matrices, in which $b_{ii} = 1$, $i = \overline{1, q-1}$; $c_{qq} = 1$; $d_{ii} = 1$; $i = \overline{q+1, n}$, and all other elements of the matrices identically equal zero, $\alpha$ and $W^0$ are given initial vectors.

We shall study problem (1) in assuming the fulfillment of the following conditions.

*Condition* 1°. $A(x), h(x) \in C^\infty[I]$.

*Condition* 2°. The spectrum of the matrix $A(x)$ is real and satisfies the conditions

$$
\begin{aligned}
\lambda_1(x) \equiv \dots \equiv \lambda_p(x) < \lambda_{p+1}(x) < \dots < \lambda_{q-1}(x) < \lambda_q(x) <&\\
< \lambda_{q+1}(x) < \lambda_{q+2}(x) < \dots < \lambda_{q+s}(x) \equiv \dots \equiv \lambda_n(x),&
\end{aligned} \tag{2}
$$

where $\lambda_i(x) \neq 0$, $i = \overline{1, q-2} \cup \overline{q+2, n}$, $x \in I$.

Unstable elements of the spectrum of the matrix $A(x)$ are representable in the form

$$
\begin{aligned}
\lambda_{q-1}(x) = x\widetilde{\lambda}_{q-1}(x), \quad \lambda_q(x) = (x - x_0)\widetilde{\lambda}_q(x), \quad \lambda_{q+1}(x) = (x-a)\widetilde{\lambda}_{q+1}(x),\\
\widetilde{\lambda}_m(x) < 0, \quad m = \overline{q-1, q+1}, \quad 0 < x_0 < a.
\end{aligned} \tag{3}
$$

From the structure of $\operatorname{Sp} A(x)$ one can see that the solution of nondegenerated vector equation

$$
-A(x)\omega(x) = h(x) \tag{4}
$$

in the general case has discontinuities of second genus at the points $x = 0$, $x = x_0$, and $x = a$.

The objective of the present article consists of

1) the construction of a sufficiently smooth solution of a singularly perturbed equation (further termed as SPE) (1) for all $x \in I$ and sufficiently small values of the parameter $0 < \varepsilon < \varepsilon_0 \ll 1$;

2) the proof of the fact: For constructed solution of problem (1) on a certain compact of segment $I$, which does not contain the points $x = 0$, $x = x_0$, and $x = a$, the following limit equality takes place

$$
\lim_{\varepsilon \to +0} W(x,\varepsilon) = \omega(x),
$$

where $\omega(x)$ is the solution of the vector equation (4).