Ice flow line modeling and ice core data interpretation: Vostok Station (East Antarctica)

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Abstract: This work, originally based on a series of the authors’ publications [101, 102, 104, 112, 130, 131], considers general questions of ice-sheet flow modeling as related to ice core records interpretation. It reviews the previous results and, using new geographical, geophysical and glaciological data, continues the study aimed at solving the twofold problem of ice core age dating and paleoclimatic reconstructions from the isotopic content measurements in the deep ice cores from Vostok Station located in central East Antarctica, above the vast subglacial lake. The principal idea of the paper is to develop a general approach to past climate investigation by means of an improved thermo-mechanical ice flow line model, involving a wide spectra of supplementary data such as borehole-temperature, radio-echo-sounding reflection layers, and air-bubble measurements.

Key words: ice sheet, ice core records, ice flow line modeling, ice age dating, past climate change

1. Introduction

Numerous evidences confirm a correlation between hydrogen δD (and oxygen δ18O) stable isotope content in solid precipitation and local temperature $T_i$ of atmospheric moisture condensation [11, 12, 18, 27, 50, 133] which, in central Antarctica, is considered to be the cloud temperature at the top of the inversion layer. However, theoretical predictions of the deuterium/inversion temperature transfer function are rather uncertain, and the estimates of its temporal slope $C_T$ generally do not agree with the observed present-day spatial gradients [21, 39, 45, 131]. Obviously, additional independent information on past climate changes is needed to further elaborate and constrain the procedure of processing the unique isotopic signals. This question is closely linked to that of ice core age dating. Actually, simultaneous solution of both problems permits supplementary paleoclimatic analysis of borehole temperature profiles. The non-stationary temperature field in the central part of the Antarctic ice sheet with extremely low accumulation rates remembers [1, 94, 107] recent Milankovich cycles of the past surface temperature variations (geophysical metronome) which, being periodically extended and correlated to the isotopic record, determine [101, 104, 106, 113] the depth-age sequence of the major climatic events (temperature maximums and minimums), i.e. geophysical metronome time scale (GMTS). Furthermore, theoretical investigations and computational experiments show that ice flow line modeling in two-dimensional (2-D) approximation becomes a useful tool for ice age predictions in central parts of an ice sheet if the model parameters are constrained by a priori chronological information [74, 75, 137]. From this point of view, GMTS can be considered [3, 34, 35, 112] as an important and statistically significant set of the so-called orbitally tuned age markers (control points). Another independent source of age markers may be provided for future studies by the local insolation proxy properties of atmospheric air trapped in ice [6, 90]. Once the ice age-depth relationship is introduced, fitting the 2-D thermo-mechanical simulations to the borehole temperature measurements reliably constrains the transformation of the ice core isotope time series into the history of ice-sheet surface temperature ($T_s$) variations [101, 104, 113, 130]. However, the above general strategy of paleoreconstructions is not complete. The borehole thermometry does not directly constrain the inversion (condensation) temperature $T_i$ which determines the water-vapor equilibrium pressure in clouds conventionally correlated [97] in the central Antarctica to solid (snow) precipitation. Hence, past temporal variations of the ice accumulation rate $b$, the principal climatic input of the ice-sheet dynamics, remain highly uncertain. The use only of a local set of the ice age-depth correlation points for tuning a multi-parameter ice-flow line model does not account for geographic peculiarities of environmental conditions and does not permit simulation of a realistic spatial flow pattern. This is especially important when, as in the case of the Vostok ice core, the drilling site is not a dome summit, but is located relatively far from the ice divide above the vast subglacial lake [46]. The situation can be substantially improved if the ice core data on geometrical properties of air inclusions are involved in the paleoclimatic studies. Based on a self-similarity of dry firn structures at pore closure [30], a new constraint on the past accumulation rates was proposed in [55] and has been later discussed in [124]. The original conclusion is that the total bubble number density is
inversely proportional to the mean ice-grain volume at
the close-off and, thus, due to normal grain growth in
snow and firn, must be related to the local climatic
conditions \((b)\) and \(T_s\) of ice formation. Recent studies
[103, 105] of the snow/firn densification process result
[56] in developing an explicit semi-empirical equation
which links relative past accumulation rates along the
flow line to the bubble number measurements and the
ice-sheet surface temperatures. Furthermore, extensive
radio-echo sounding (RES) observations were carried
out from the ice-sheet surface by the Polar Marine
Geosurvey Expedition (PMGE) in the vicinities of
Vostok Subglacial Lake (VSL) in 2004-2006. In
particular, the bedrock relief and distinct isochronous
(reflection) layers with the ice age up to ~220 kyr were
detected in RES profiling over the 106-km traverse
along the ice flow line upstream of Vostok Station. The
described complex of experimental and theoretical
works for the first time has provided various
palaeoclimatic, glaciological, and geophysical data from
Vostok area to attempt a joint self-consistent analysis
based on the ice flow line modeling and aimed at
simultaneous solving the twofold problem of ice-core
date aging and palaeoclimatic reconstructions from ice-
core isotopic records.

Aside from ice-sheet dynamics and formation of
palaeoclimatic signals, the study of heat transfer along
ice flow lines starting from Ridge B and passing across
the lake and Vostok Station is of primary importance for
our understanding the gas (air) and water exchange
between the ice cover and the subglacial water basin, as
well as for investigation of biological processes and
possible origins of microbial life in the lake (e.g. [77,
117]). Non-stationary temperature fields determine the
ice accretion and melting rates at the ice - water
interface [76, 77, 109, 125, 130] and, as a consequence,
lake salinity and the rates of air accumulation in the lake
in the forms of dissolved air and air hydrates [54, 68].
Temperature conditions along ice particle trajectories in
the glacier directly control the air hydrate crystal growth
[108, 132] and the state of micro-organisms transported
through the glacier from its surface towards the lake.

With this in mind, we, first, review and compare after
[131] different approaches to palaeoclimatic
interpretation of ice core isotopic records, borehole
temperature profiles and discuss existing uncertainties
in ice age datings of different origins at Vostok. Then,
in a continuation of [100, 130], an improved
sophisticated ice flow line thermo-mechanical model is
elaborated for simulating the ice sheet dynamics and
temperature field along the Vostok flow line with
account of the ice deformation within the bedrock
- subglacial lake transition zone and the ice accretion at
the glacier bottom over the lake. Recent RES
observations, updated geographic, geophysical and
glaciological data are summarized. Supplementary
information about past accumulation rates deduced from
the bubble density measurements [56] is also described.
Finally, the general strategy and results of the newly
performed computational experiments are discussed to
present the re-examined glaciological time scale for the
Vostok ice core and resultant palaeoclimatic
reconstructions.

2. Paleoclimate in the Antarctic ice memory.

Previous studies

2.1 Isotope content of ice

Although we conventionally refer to the correlation
between stable isotopes in solid precipitation in central
Antarctica and local inversion (condensation)
temperature \(T_s\), it should be noted that the isotopic
content of deep ice cores can differ from that of the
fresh snow due to the surface snow metamorphism and
mass exchange with the atmospheric moisture. The
sublimation and recrystallization effects in snow and
firn (e.g. [28, 49, 115]) are primarily controlled by the
near-surface glacier temperature \(T_s\), which, in central
Antarctica, is significantly lower than \(T_s\) due to the
developed inversion strength. As a result, the stable
isotope content in ice cores corresponds to a certain
apparent temperature, which, in general, may not
coincide with any of the instrumentally measured
values. Nevertheless, it is still assumed that the
temperature fluctuations themselves deduced from the
isotopic content variations do not differ much from
those of the effective temperature of the atmospheric
moisture condensation. The detailed analysis of the
present-day isotope-temperature correlations based on
meteorological observations, measurements in pits, and
borehole thermometry for the recent 50 years [21, 23,
24, 59] confirms this conclusion. Importance of the
deuterium excess \(d_{ex} = \delta D - 8\delta^{18}O\), as a derived
paleosignal, and its relation to thermodynamic
conditions of moisture formation in the evaporation
zone, e.g., source temperature \(T_v\), have been
demonstrated in [38, 42, 43, 79]. A complete set of the
ice core isotopic data from the deep borehole at Vostok
Station to a depth of 3310 m and its detailed analysis are
presented in [135]. Fig.1 depicts results of these
measurements together with the parabolic mean-square
spline approximation of the ice isotope content
fluctuations versus depth. Substantially more detailed
experimental studies on the Vostok ice core deuterium
content to 3350 m were performed and summarized
earlier in [78].

The advantage to employ simultaneously both
isotopic signals, \(\delta D\) and \(d_{ex}\), in palaeoclimatic
reconstructions and the necessity to deduce past
temperature variations at a site of solid (snow)
precipitation together with those in the moisture-
evaporation zone was shown in [14, 134]. Linear
relationships were proposed to determine the relatively
small temperature perturbations:

\[
\begin{align*}
\Delta \delta D &= \gamma_T \Delta T_s - \nu_T \Delta T_v + \nu_O \Delta ^{18}O_v, \\
\Delta d_{ex} &= -\beta_T \Delta T_s + \beta_T \Delta T_v - \beta_O \Delta ^{18}O_v .
\end{align*}
\]  

(1)
Here $\delta^{18}O_w$ is the ocean water oxygen isotope content, $\Delta$ means the deviation of the characteristics from their contemporary values; $\gamma_i$, $\gamma_m$, $\gamma_{rm}$, $\beta_i$, $\beta_m$, $\beta_{rm}$ are the regression coefficients, which are introduced as positive values and describe the climatic links of the site under consideration with the processes of atmospheric circulation.

At a given correction for the isotopic content of sea water $\Delta\delta^{18}O_w$ [4, 36, 78, 123], the system of the algebraic equations (1) allows to calculate variations of the climatic parameters $\Delta T_i$ and $\Delta T_m$ from the isotope content of ice $\Delta\delta D$ and $\Delta\delta^18O$. If, as a first approximation, a proportionality of the temperature fluctuations is assumed [48], i.e. $\Delta T_m \approx r_{\alpha_i} \Delta T_i$, then the first of equations (1) reduces to the relationship [41] traditionally used in paleoreconstructions for estimation of the inversion temperature variations

$$\Delta T_i = (\Delta\delta D - \gamma_m \Delta\delta^{18}O_w)/C_T, \quad C_T = \gamma_i - \gamma_{\alpha_i} r_{\alpha_i}. \tag{2}$$

At the same time, in accordance with [97], the amount of precipitation (ice accumulation rate $b$) in Antarctica is correlated with the water vapor saturation pressure in clouds and can be calculated after [94, 95] as

$$b = b_0 \tilde{F}(s) \exp(\eta_b \Delta T_i), \tag{3}$$

where $b_0$ is the present-day value of the accumulation rate at the site under consideration ($s = s_0$), $\tilde{F}(s)$ is the normalized spatial distribution of the accumulation rate along a reference ice flow line with the distance $s$ counted from the ice divide.

The coefficient $\eta_b$ in Eq. (3) can be expressed [95] via the present-day condensation temperature (the cloud temperature $T_0$ at the top of the inversion layer) $\eta_b = 6148.3/(273.15 + T_0)\frac{1}{2}$, and at $T_0 = -38.0\pm0.6^\circ C \approx 0.112^\circ C^{-1}$. Close estimates of $\eta_b \sim 0.10$-$0.14^\circ C^{-1}$ follow from the geographical dependence of the accumulation rates on the surface content of the surface snow layer in East Antarctica along the traverses "Mirny Observatory - Vostok Station" [21, 22, 53] and "Syowa Station - Dome Fuji" [114].

The large inversion strength, $T_i - T_s$ over the Antarctic Plateau generally implies the necessity to develop special approaches for reconstruction of the surface temperature variations $\Delta T_i$ in the past. Based on the present-day spatial distributions of the surface and inversion temperatures, Jouzel and Merlivat [42] proposed a linear proportionality between $\Delta T_i$ and $\Delta T$: $$\Delta T_i = \Delta T/C_i. \tag{4}$$

Eqs. (1)-(4) form a theoretical basis for paleoreconstructions from the ice core isotopic data stored in the Antarctic ice sheet memory. The principal problem of their practical employment is in reliable constraining of the coefficients in this paleoclimatic model. In particular, the reviews [14, 48, 102, 134, 135] are devoted to the validation of Eqs. (1). A summary of results obtained in these publications is presented in Table 1. The simplified Rayleigh-type model for the atmospheric moisture condensation and isotopic fractionation in air masses moving along a single trajectory [10, 18, 26, 38, 42, 69] was used in [14, 134] to calculate the isotopic composition of precipitation in Central Antarctica. Although this model omits the evaporative recharge of the air mass by moisture from the ocean, the derived values of the $\gamma$ and $\beta$ coefficients (variant V-1) were, partly, confirmed by simulations on the basis of global atmospheric circulation models (GCM) and were compared with the direct geographical observations. In [102], the Rayleigh scheme of the isotopic fractionation was re-examined to take additionally into account the feedback between the isotope content of the moisture in the air leaving the low-altitude evaporation zone and the influx of the less moist and isotopically depleted air coming to the source area. It is important that the computations (variant V-2) revealed a substantially higher sensitivity (coefficient $\beta$) of the deuterium excess $\delta D$ to the condensation temperature $T_i$ at a precipitation site. A new "intermediate" model for the formation of the isotopic composition in precipitation in Antarctica was proposed by Kavanaugh and Cuffey [48] as a generalization of [25, 32, 47]. This model includes the turbulent convective mixing in the longitudinal direction and the evaporation (recharging) from the ocean. However, the vertically integrated uniform profiles of all important characteristics (temperature, humidity, isotopic ratios etc) were assumed, implying infinitely fast altitudinal air and moisture mixing. Thus, the two simplified approaches [48] and [102] to the description of the isotopic fractionation in the global hydrological cycle can be considered as limiting scenarios with respect to the vertical water-vapor transfer effects. Computational experiments performed by Kavanaugh and Cuffey [48]
to study the general sensitivity of the isotope content of ice deposits in the Vostok Station area to the global temperature changes did not result in unique estimates for the coefficients $\gamma_i$, $\gamma_o$, and $C_T$ in Eqs. (1) and (2) (variant V-3, Table 1) and demonstrated a strongly dependent on possible moisture exchange regimes. Systematically lower estimates of $\gamma_i$ and $\gamma_o$ at intermediate values of $\beta_i$ were obtained in this case in comparison with the variants V-1 and V-2, while, as emphasized in [14, 48], the coefficients $\gamma_o$ and $\beta_m$ were reliably determined. It should be noted after [45] that, in accordance with this work, $\gamma_o$ in Eqs. (1) and (2) is equal to 4.6 instead of 8 traditionally assumed earlier.

The difference in the estimates of the principal coefficients $\gamma_i$, $\gamma_o$, and $\beta_i$, $\beta_m$ is rather significant and, as shown in [102], can lead to the substantial uncertainty in paleo-reconstructions. Fig. 2a evidently illustrates this conclusion and demonstrates how different the deduced fluctuations of $\Delta T_i$ and $\Delta T_o$ can be, depending on a chosen combination of the coefficients in Eqs. (1).

Unfortunately, numerous attempts to use GCMs were focused [21, 45] mainly on Eq. (2) describing temporal variations of the condensation temperature in order to investigate the difference of the combined coefficient $C_T$ from the corresponding spatial (geographic) isotope/inversion temperature slope. In particular, for the Vostok area conditions, a 30% reduction of $C_T$ in comparison with its geographic analogues can not be excluded [39, 45]. Such deviations are also in agreement with the general relations (1). For example, if the contemporary latitudinal distributions of isotopes are formed by air masses coming from one source, at $\Delta T_o = 0$, the spatial $C_T$ slope in Eq. (2) should be identified with $\gamma_i$. On the other hand, in accordance with the predictions of isotopic fractionation models (see variants V-1, 2, 3, Table 1), the ratio $C_T$ between the temporal variations of $\Delta \delta D - \gamma_o \Delta \delta^{18}O_m$ and $\Delta T_i$ for $r_m \sim 0.5-1$ in Eqs. (2) will be 15-35% lower.

Available meteorological observations and special measurements in pits in vicinities of Vostok Station were analyzed and summarized in [21]. The deduced statistically valid regression coefficients of the correlation between the seasonal condensation temperature oscillations and the isotopic composition of precipitation ($C_T$) during the year 2000, as well as between long-term (over 50 years) annual inversion temperature variations and the air temperature near the ice sheet surface ($C_i$) are presented in Table 1. Let us note that seasonal changes in the inversion temperature and the surface air temperature give almost a two times lower value of $C_i$ which agrees with the earlier estimate [80]. This difference is explained by the seasonal character of the inversion strength [21].

Geographic data on the present-day distribution of the isotopic content in surface snow over Antarctica and the correlation between the isotopes and the surface temperature, including measurements [15, 62], were analyzed in [21, 22]. For East Antarctica, the ratio between the deuterium content variations and the near-surface air temperature was determined to be $C_i/C_T \approx 6.4 \pm 0.2^{\circ} C^{-1}$ and is in close agreement with the value of $6.04^{\circ} C^{-1}$ from [62] and with the estimate [114] obtained for the oxygen isotopes corresponding to $C_i/C_T \approx 6.8 ^{\circ} C^{-1}$ for deuterium. According to [42], the spatial relationship (4) between the inversion and surface temperatures in Antarctica is characterized by the coefficient $C_i = 0.67$. Consequently, the traditionally used estimate of $C_T \approx 9^{\circ} C^{-1}$ [78] directly follows [41] from [62]. However, as shown in [21], the inversion temperature is close to that of the atmospheric moisture condensation in clouds only in the central regions, and spatial variations of the condensation temperature are described by noticeably lower values of $C_i \approx 0.45-0.52, \approx 6.8^{\circ} C^{-1}$.
and temperature in Antarctica can not be directly used in paleoconstructions of climatic time series even in the simplified procedure (2)-(4). Totally, the geographic estimates of the slope \( C_T \approx 9-13\% / ^\circ C \) are not in contradiction with the theoretical predictions of the coefficient \( \gamma \approx 7-11\% / ^\circ C \) in Table 1. However the observational data do not specify the calculated values. At the same time, as can be seen from Table 1, the meteorological observations of the short-term changes in the inversion temperature and the isotopic composition in precipitation at Vostok Station [21], in spite of their high uncertainty, give the estimates of the temporal slope in Eq. (2) close to the model results \( C_T \approx 5-7.5\% / ^\circ C \) [48]. Furthermore, it follows from [21] that the difference between the temporal variations of the inversion temperature (apparent condensation temperature) and the surface temperature at Vostok Station (see Table 1) is substantially less \( C_i \approx 0.7-0.8 \) in comparison with their spatial distributions \( C_i \approx 0.67 \) or even 0.45-0.52).

### 2.2 Ice sheet temperature field

The reviewed studies support very high informativeness of the ice core isotopic records form deep boreholes drilled in central parts of Antarctic ice sheet. However, it becomes clear that additional independent data on the past climate changes should be involved in the interpretation of these unique paleoclimatic signals to further validate and calibrate the transfer functions (1)-(4). This task does not exhaust the whole problem of paleoconstructions as a procedure of deriving the past temperatures and precipitation. Another closely related question is the ice core age dating. A simultaneous solution of these two problems permits supplementary paleoclimatic analysis of borehole temperature measurements [104, 106].

Let us discuss this point in more details. The first computational experiments [1, 16, 94] showed that borehole temperature profiles in the large ice sheets on our planet contain the information about past temperature fluctuations at the glacier surface. Further studies [13, 17, 37] of the heat transfer processes in the central regions of Greenland ice sheet with relatively high accumulation rates and moderate ice thickness allowed directly to infer the detailed surface temperature variations over the recent 25-30 kyr from the temperature surveys in deep boreholes and made it possible to analyze the relationship between the temperature and the isotope content of ice. The far past history was "forgotten".

As shown in [101, 104, 106, 107], in contrast to Greenland, the non-stationary temperature field in the centre of Antarctic ice sheet with extremely low accumulation rates and maximum ice thickness remembers distinguishable local temperature perturbations, induced by the recent astronomical Milankovich cycles with the eccentricity, obliquity, and precession periods of \( t_1 = 100, t_2 = 41, \) and \( t_3 = 23, t_4 = 19 \) kyr dominating in the Pleistocene climate in central Antarctica [4, 36, 141]. However, more or less

Figure 2: Uncertainty of paleotemperature reconstructions. (a) Variations of the inversion temperature \( \Delta T_i \) and moisture-source temperature \( \Delta T_w \) deduced from Eqs (1) at the mean values of the coefficients for variants V-1 – V-3 in Table 1 (curves 1-3, respectively), the shaded area is the uncertainty of the inversion temperature reconstructions in V-3 [48]. (b) Variations of the inversion and surface temperatures \( \Delta T_i \) and \( \Delta T_s \) from Eqs (2) and (4) at the mean values \( C_T = 6.2 \% / ^\circ C \), \( C_i = 0.75 \) for meteorological observations [21] and ice flow modeling [112] (curves 1), for geographic estimates \( C_T = 9 \% / ^\circ C \), \( C_i = 0.67 \) [41,42] (curves 2) and from Eqs (2) and (6) after [3] at \( C_T = 7.7 \% / ^\circ C \), \( C_i C_T = 3.8 \% / ^\circ C \) with \( \alpha_p = 0.18 \) (curves 3) and \( \alpha_p = 0 \) (curve 4).

resulting in substantially higher estimates of \( C_T \approx 13 \% / ^\circ C \) for the condensation temperature.

Thus, the correlations between contemporary spatial distributions of the isotope composition of surface snow
reliable information about the details of short-term temperature fluctuations in the past on time scales of 3-5 kyr is lost. This restricts substantially the applicability of direct inverse methods for paleoclimatic reconstructions on the basis of borehole temperature measurements alone [72, 101]. Yet, representing, in accordance with the Milankovich climate theory, the surface temperature fluctuations as a superposition of harmonic oscillations of the fixed frequencies \( \omega_j = 2\pi t_j \) (\( j = 1, \ldots, 4 \)),

\[
T_s(t) = T_{so} + \sum_{j=1}^{4} \left[ A_j \left( \cos(\omega_j t) - 1 \right) - B_j \sin(\omega_j t) \right],
\]

(5)
it becomes possible to find [3, 101, 104, 106, 113] the contemporary ice sheet surface temperature \( T_{so} \) and the cosine and sine amplitudes \( A_j \) and \( B_j \) of the four Milankovich cycles by minimizing the standard deviation (SD) between simulated and measured temperature profiles. Here \( t \) is the time counted from the far past (\( t < 0 \), and \( t = 0 \) is the present moment).

Various borehole thermometry data of different quality and observational depths were used in the above cited papers. Despite noticeable deviations of inferred amplitudes, the geophysical metronome (5) reliably reproduced (with accuracy of \(-\approx 2 \) kyr) the ages of the major climatic events (temperature maximums and minimums) at the glacier surface. From this point of view, it is of principal importance that the same paleoclimatic extrema are clearly distinguished in the smoothed variations of the deuterium content in ice cores versus depth (Fig. 1). Consequently, although the absolute temperature transitions between different climatic stages are not precisely determined, the ice age dating seems to be rather reliable. The resulting sequence of the depth-age correlation points was called [104] the geophysical metronome time scale (GMTS). The age markers uniformly distributed versus depth can be used to deduce an estimate for the coefficient \( C_t \) in Eq. (3) which determines the accumulation rate, the principal climatic input of ice sheet flow models. The value of \( C_t \) must be in agreement with GMTS on average, resulting in the minimum standard deviation of the simulated (so-called glaciological) ice core time scale from the given set of control points. Preliminary fitting was performed on the basis of a simplified ice flow model [3, 34]. The obtained respective values of \( C_t \approx 7.7\pm1.1 \) and 6.7\%°C\(^{-1}\) do not contradict with the other estimates \([21, 48]\) in Table 1. However, the coefficient \( \eta_2 \) in Eq. (3) was determined [95] only approximately, and its error is automatically included into the deduced value of the \( C_t \) parameter.

Once the ice core time scale is introduced, the borehole temperature measurements can be employed [94] for calibration of Eqs. (1), (2) and (4) which transform the isotopic time series into temporal variations of the ice sheet surface temperature \( T_s \) as the principal climatic factor of the model input. For the simplified relations (2) and (4), only \( T_{so} \) and the product \( C_C C_t \) (or \( C_t \)) must be tuned. However, the analysis of the high-precision temperature profile measured by Yu. Rydvan [106] at Vostok Station in 1988 to a depth of 1900 m showed [104] that, at least, the recent climatic fluctuations \( \Delta T \) contained a selectively amplified (supplementary) precession signal \( \delta_\omega \) which could not be reproduced simply by scaling of the inversion temperature variations \( \Delta T \) in Eq. (4). To account for this, although small, correction, a generalized form of Eq. (4) was proposed

\[
T_s = T_{so} + \Delta T / C_t + \delta_\omega(t),
\]

(6)

where \( \alpha_p \) is the scaling factor of the precession component of the local geophysical metronome (5) [3, 101, 104, 113].

The best-fit values of the product \( C_C C_t \) and the \( \alpha_p \) coefficient were first inferred [104] from Rydvan’s temperature profile [106] and in two limiting cases were found to be \( C_C C_t \approx 4.8\pm0.3 \) and 4.2\%±3.3\%°C\(^{-1}\), \( \alpha_p \approx 0.29 \) and 0.14, respectively. Less accurate temperature profile down to a depth of 3590 m was used in [101, 113] and lead to lower estimates of \( C_C C_t \approx 3.3 \) and 2.9\%°C\(^{-1}\) at \( \alpha_p \approx 0.24 \) and 0.31. Preliminary results of the new continuous temperature survey performed by R. Vostretsov at the end of 1999 to a depth of 3620 m in the record borehole at Vostok Station (almost two years after the drilling operations had been stopped) were analyzed in [3]. They gave intermediate values of \( C_C C_t \approx 3.8\pm0.1\%°C\(^{-1}\) and \( \alpha_p \approx 0.18 \). In all the above cited papers, an approximate quasi-one-dimensional heat transfer model [101, 106, 107] was used for simulating the ice-sheet temperature field in central Antarctica. In addition, it becomes clear that the inferred values of the tuning coefficients are substantially influenced by accuracy of temperature measurements. The bounds of the deduced parameters in Eqs. (6) are presented in Table 1. They are in agreement with the estimate of \( C_C C_t \approx 3.6\pm1.0\%°C\(^{-1}\) inferred in [59] from borehole temperature observations in a 100-meter surface layer at Vostok Station over the last 50-year period. It can easily be seen that in all these cases the use of the direct measurements \([21]\) of the \( C_t \) coefficient or its geographic analogue [42] leads to the calculated values of \( C_t \) comparable to other independent estimates of this parameter in Table 1 except for the much higher geographic slopes. It should be mentioned that these estimates of \( C_t \) do not take into account the supplementary precession signal \( \delta_\omega \) in the relationship (6) which slightly increases the amplitude of temperature oscillations at the ice sheet surface in comparison with the simplified equation (4).

Various limiting cases of the inversion and surface temperature fluctuations \( \Delta T \) and \( \Delta T \) reconstructed from the ice core deuterium record (see Fig. 1) on the basis of Eqs. (2), (4) and (6) calibrated by the temperature measurements at Vostok Station are shown in Fig. 2b.
These results agree with the calculations of $\Delta T_i$ from Eqs. (1) in Fig. 2a, although they also demonstrate a significant diversity. In particular, the decrease in the inversion temperature (effective condensation temperature) determined by meteorological observations and the change in the ice sheet surface temperature inferred from the borehole temperature profile for the LGM at Vostok Station are 30-50% larger than the corresponding estimates based on the geographic data. The impact of the precession signal $\delta_p$ in Eqs. (6) is revealed (see dotted curve in Fig. 2b) in more pronounced (by 2-2.5°C) climatic maxima in the surface temperature.

3. Age of the Vostok ice core. Uncertainties and problems

3.1 Ice age datings of different origins

There is no universal and/or standard procedure to determine depth-age relationships in polar ice sheets at sites of deep drilling. The latter question is of primary importance for the deep ice core retrieved from the Antarctic ice sheet at Vostok Station [78]. Possible applicability of sophisticated 2-D and 3-D thermomechanical ice flow models [95, 96] for solving this problem significantly suffers from inevitable uncertainties in initial and input data, mainly in glacier bottom conditions and past accumulation rates. This was the principal obstacle limiting the accuracy of earlier simulations and ice age predictions by ~15-20 kyr. Nevertheless, theoretical studies and computational experiments [74, 75, 137] show that the modeling of ice sheet dynamics becomes a useful tool for the ice age prediction in central areas of large ice sheets if a priori chronological information is used to tune the model parameters. Not being free from their own specific errors, different sources of age markers and dated time series can, yet, be considered as reliable constraints if all together they deliver statistically valid and independent estimates of ice age at various depth levels. The inverse Monte Carlo sampling method (e.g. [70, 71, 128]) is especially helpful in this case [74] to fit the ice-sheet model on average, uniformly versus depth, although without putting much weight on local (high-frequency) fluctuations of the simulated depth-age curve caused by uncertainties in environmental conditions, reconstructed ice accumulation and other paleoclimatic characteristics. From this point of view, as a first approximation, even simplified ice flow models (e.g. [3, 34, 35, 110, 112]) may be quite appropriate to incorporate the principal laws of ice-sheet dynamics into the ice core dating procedure.

Among different depth-age relationships developed for Vostok, the geophysical climatome time scale (GMTS) [104, 106] extended in [3, 101, 113] to the maximum depth 3350 m of the Vostok ice core isotope record covering four interglaciations represents the so-called orbitally tuned chronologies. As explained in section 2, it is based on the correlation of the isotopic temperature signal with the geophysical metronome, i.e. Milankovich components of the local surface temperature variations in the past inferred from the borehole temperature profile. Possible errors and uncertainties of GMTS were discussed in [101, 104], and its overall average accuracy was estimated as ±3.5-4.5 kyr. Orbital ice age control points used in [74, 75] as model constraints coincide (to within ±2 kyr) with the corresponding GMTS ages.

Another paleotemperature-proxy signal spanning over more than 500,000 years is available from the calcite core in Devils Hole (DH), Nevada [139, 140]. The principal advantage of this $\delta^{18}O$ record is that the dense vein calcite provides a material for direct uranium-series dating [63] with the relative standard errors of about 1.5-2%. In addition, the signal may be influenced by different hydrological factors. In particular, a systematic underestimation of the determined ages on the order of several (two) thousands of years may take place due to the ground-water travel time through the aquifer [51, 139]. Nevertheless, the correlation of the Vostok ice core deuteration-depth signal with the Devils-Hole record (see U.S. Geological Survey Open-File Report 97-792 at http://pubs.water.usgs.gov/ofr97-792) can also be considered as an independent approach to dating long paleoclimate archives from central Antarctica [51].

Note, that we do not discuss here a possibility (e.g. [7, 123]) to employ various gas studies of Vostok ice cores for relative dating (correlation) with other Antarctic, Greenland or deep-sea core climatic records, because of inevitable additional errors which arise in this case from the uncertainty in the gas-ice age difference preventing better constraints on the ice age.

3.2 Glaciological and average time scales

The best-guess glaciological time scale simulated in [75] was mainly based on the RES data by Siegert and Kwok [118] and is designated as GTS-I after [131].

The principal goal of [112] was (1) to combine the GMTS and DH sources of chronological information (70 age markers of climatic events totally) in order to constrain ice flow model parameters, reducing to minimum systematic errors in the glaciological time scale based on simulations of ice sheet dynamics in the vicinities of Vostok Station and (2) to deduce an average age-depth relationship consistent with different approaches to ice core dating within the upper 3350 meters. For the grounded part of the ice sheet, the reference flow line passing through Vostok Station was drawn perpendicular to the surface elevation contours [119], and the ice flow tube configuration was approximately determined after [117, 118, 127]. Over the lake, the flow line was continued in accordance with the observations [5, 125]. A monotonically increasing spatial distribution of the normalized mass balance $B(s)$ in Eq. (3) was schematically taken as similar to the smoothed accumulation rate profile along the traverse to Vostok [53] with account of the mass-balance enhancement factor 1.65 estimated [40] for Ridge B at the location of the Dome B ice core. The present-day ice-sheet thickness at Vostok was assumed
to be 3755 m after [64]. The bounds for the mean recent accumulation rate at Vostok were determined on the basis of [2, 22, 24].

Tarantola's theory [128] and the Monte Carlo (random walk) sampling procedure [70, 71] were employed in [112] to solve the inverse problem and to find the optimal model parameters among which the principal ones were the present-day accumulation rate $b_0$ and the deuterium/inversion temperature slope $C_T$ in Eqs. (2) and (3). The modelled best-guess ice age-depth distribution is called GTS-II. Its standard deviations from GMTS and DH age markers were estimated as $-4.5$ and $7$ kyr, respectively. In general, GTS-II can be considered as another source of ice age estimates with minimum systematic error, though containing its own independent short-term distortions which are the attributes of the model imperfections and uncertainties of the geographic input. This finally suggested construction of an average time scale for the Vostok ice core down to a depth of 3350 m based on GTS-II and linear interpolations of GMTS and DH age-depth correlation points. The averaging procedure was performed iteratively with the weights taken inversely proportional to standard deviations of the basic chronologies from the resulting smoothed average curve. The highest weighting coefficient (accuracy) of 44% was determined for the best-fit glaciological time scale GTS-II; the weights of 38 and 18% were found for GMTS and DH age markers, respectively, in accordance with their statistical validity. The average time scale is consistent with all three basic age-depth relationships within the standard deviation bounds of $\pm 3.6$ kyr which correspond to the upper estimate of the age uncertainty. Actually, the weighted mean standard deviation of the averaged chronologies (i.e., $\pm 2.2$ kyr) is thought to be a more reliable measure of the accuracy of the resulting mean ages. Although one may argue that the simplified ice flow model, high uncertainties in the environmental conditions and the use of DH isotope time series do not give any advantage to the average time scale, the latter age-depth relationship, still, can be regarded as a common reference basis for intercomparison of different datings for the Vostok ice core.

The tuning of the ice sheet flow model was performed in [112] for the climatic input described by Eqs. (2) and (3), and, at the correction factor of $\gamma_m = 8$ for the sea-water isotopic composition, the best-fit values of the temporal isotope/inversion temperature slope were inferred as $C_T \approx 6.0-6.3^\circC^{-1}$ with the standard deviation of about $0.6^\circC^{-1}$. The use of the re-examined value of $\gamma_m \approx 4.6$ in Eq. (2) instead of 8 reduces the estimate of $C_T$ by $-0.4^\circC^{-1}$ without any influence on the ice dating. The obtained result agrees with the other data presented in Table 1. The best-fit present-day accumulation rate was found at its lower bound $2.15$ cm yr$^{-1}$ and matches with the mean value $2.23 \pm 0.03$ cm yr$^{-1}$ calculated for the recent 190 years from the depths of the Tambora eruption layer measured at Vostok in 9 shallow boreholes and deep pits [24].

![Figure 3: Comparison of the average time scale for Vostok ice core [112] (zero line) with the best-guess glaciological time scales GTS-I [75], GTS-II [112] and GT-4 [78] (curves 1-3, respectively). The shaded area is the confidence interval of the running averaged ages. Filled and open circles are the GMTS and DH age markers, triangles are the control points used in [75].](image)

The average time scale (zero reference line) is compared in Fig. 3 with the previously used glaciological time scale GT-4 [78] and the best-guess age-depth relationships GTS-I and II developed through the ice sheet model constraining in [75] and [112]. The GMTS and DH control points are shown by empty and solid circles, respectively. The shadowed area in the figure represents the confidential interval for the average time scale with the boundaries determined as the standard deviations of the three basic chronologies from the mean ages. As can be seen, an obvious progress has recently been achieved in ice core age dating at Vostok. It is also important that the above cited papers [75, 112] used similar modeling approaches and inverse methods. The standard deviation between GTS-I and II is about $3.3$ kyr on average and is comparable with the estimated errors of the average time scale. However, local discrepancies between GTS-I and GTS-II ages still reach 7-8 kyr and are mainly caused by differences in the initial geographic and paleoclimatic data. New studies and special sensitivity tests [130, 131] additionally confirm that the reliability of ice core age dating and paleoreconstructions depend substantially on accuracy of thermo-mechanical modeling and quality of input (environmental) information.

4. Vostok flow line modeling

4.1 Ice flow tube geography and RES data.

As mentioned above, environmental ice sheet flow conditions previously used in [75, 112, 130, 131] were rather uncertain and in some cases even schematic.

In this study, the Antarctic ice sheet surface elevation map of the Lake Vostok vicinities (Fig. 4) is based on the ERS-1 satellite altimetric data [60, 92] adapted after
Figure 4: Surface elevation map of Lake Vostok vicinities [60, 92] adapted after [129]. 1- Vostok ice flow line (VFL); 2- ice flow tube; 3- SPRI/NSF RES route [19, 98]; 4- Lake Vostok water table [83]; 5- elevation contours with 10-m spacing.

Figure 5: Bedrock topography map of Lake Vostok vicinities after ABRIS project [82]. 1- Vostok ice flow line (VFL); 2- SPRI/NSF RES routes [20, 98]; 3- Lake Vostok water table [83]; 4- elevation contours with 200-m spacing.
Figure 6: Ground-based radio-echo time section (a) and the internal structure, surface and base topography (b) of the ice sheet over the 106-km distance along the ice flow line upstream of Vostok across Vostok subglacial lake (VSL). Reflection layers marked by open circles and numbered from L0 to L7 are used in this study for model constraining.

It has been combined with the detailed geophysical observations around the lake provided by PMGE [67, 83, 84]. The re-examined Vostok flow line (VFL) and the flow tube confined between two neighboring flow lines are drawn in Fig. 4 perpendicular to the elevation contours for the grounded part of the ice sheet and follow the ice velocity field over the lake as determined in [5, 129] and recently confirmed by [138]. We introduce the longitudinal coordinate $s$ as a distance measured from Ridge B along VFL with Vostok Station located at $s_0 = 370$ km.

Since the international BEDMAP project [65], many radio-echo sounding and seismic studies have been performed in Antarctica, in particular in the Lake Vostok area (e.g. [67, 83, 84, 125, 127]). The revised fragment of the general bedrock elevation map presented in Fig. 5 is a result of the newly launched ABRIS (Antarctic Bed Relief and Ice Sheet) project.
4.2 Ice flow description

The interaction of the ice sheet dynamics with heat transfer processes plays a special role in the vicinities of VFL passing through the vast subglacial lake. Taking account of these peculiarities, the joint problem of ice core age dating and paleoclimate reconstruction was considered in [130, 131] in the framework of the 2-D description of ice sheet movement along a fixed flow tube [86, 91]. The ice flow characteristics and the equation of convective heat transfer were presented in the boundary layer (shallow ice) approximation [31, 86, 87, 99] extended to the snow/firn compressibility effects [100] in a simplified form after [101, 107]. However, an important mass-conservation impact on vertical velocities near the western shoreline of the lake at the transition from the grounded to floating ice sheet flow pattern was neglected. Here we improve the velocity field description, using the original expressions derived in [100].

The principal assumption is made that the pattern of the spatial distribution of the ice mass balance \( b \) on the glacier surface does not change significantly with time \( t \) and the direction of ice motion is mainly determined by the bedrock relief. As a consequence, ice flow lines remain invariable, and the configuration of the fixed reference Vostok flow tube is characterized by its normalized width \( H(s) \) and current ice-equivalent

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Denotation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Snow-firn densification</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Density of pure ice, kg m(^{-3})</td>
<td>( \rho_0 )</td>
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</tr>
<tr>
<td>Surface-snow porosity</td>
<td>( c_s )</td>
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</tr>
<tr>
<td>Exponential densification factor, m(^{-3})</td>
<td>( \gamma_s )</td>
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</tr>
<tr>
<td>Ice flow model</td>
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<td></td>
</tr>
<tr>
<td>Present-day accumulation rate at Vostok, cm yr(^{-1})</td>
<td>( b_0 )</td>
<td>2.15</td>
</tr>
<tr>
<td>Mass-balance exponential factor, °C ( ^{-1})</td>
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</tr>
<tr>
<td>Flow line length, km</td>
<td>( s_0 )</td>
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</tr>
<tr>
<td>Open lake area, km</td>
<td>( s_f )</td>
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</tr>
<tr>
<td>Ridge-B highland boundary, km</td>
<td>( s_b )</td>
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</tr>
<tr>
<td>Glen flow-law exponent</td>
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<tr>
<td>Modified Glen flow-law exponent(^\dagger)</td>
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</tr>
<tr>
<td>Shear-flow-rate factor</td>
<td>( \sigma )</td>
<td>1, 0 &lt; ( s &lt; s_b ) ( 1 \rightarrow 0, s_b &lt; ( s &lt; s_f ) ( 0, s &lt; s_b )</td>
</tr>
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<td>Ice-sheet growth feed-back factor</td>
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<tr>
<td>Mean mass-balance excess(^\ddagger)</td>
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<tr>
<td>Heat transfer model</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Specific heat capacity of ice, kJ (kg °C(^{-1}))</td>
<td>( c(T) )</td>
<td>( c_0[1+\alpha_c(T+30)], )</td>
</tr>
<tr>
<td>Thermal conductivity of ice, W (m °C(^{-1}))</td>
<td>( \lambda(T) )</td>
<td>( \lambda_0[1-\alpha_\lambda(T+30)], )</td>
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<tr>
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<td>Fusion temperature at the ice-lake interface, °C</td>
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<td>Pressure depression coefficient, °C MPa(^{-1})</td>
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<td>Geothermal flux, W m(^{-2})</td>
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<tr>
<td>Thermal conductivity of rocks, W (m °C(^{-1}))</td>
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<tr>
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<td>1.2-10(^{-6})</td>
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</tr>
<tr>
<td>Present-day inversion temperature, °C ( \dagger)</td>
<td>( T_{i0} )</td>
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</tr>
<tr>
<td>Geophysical metronome amplitudes, °C [3]:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( A_1(B_1) )</td>
<td>6.28 (-2.81)</td>
<td></td>
</tr>
<tr>
<td>( A_2(B_2) )</td>
<td>5.31 (-1.86)</td>
<td></td>
</tr>
<tr>
<td>( A_3(B_3) )</td>
<td>-4.92 (2.35)</td>
<td></td>
</tr>
<tr>
<td>( A_4(B_4) )</td>
<td>-1.64 (-3.14)</td>
<td></td>
</tr>
</tbody>
</table>

\( ^\dagger \) Parameters are constrained by available data and validated in computational experiments
thickness $\Delta(s, t)$. It is also relevant to define the vertical coordinate $\zeta$ as the relative distance from the glacier base expressed in terms of the equivalent thickness of pure ice and normalized by $\Delta$.

The snow/firm and bubbly ice density $\rho$ versus depth $h$ can be presented [103, 107] as

$$ \rho = \rho_0 \left(1 - c_s e^{-\gamma_s h}\right), $$

where $\rho_0$ is the density of pure ice (constant value), $c_s$ is the porosity of the surface snow and $\gamma_s$ is the densification exponent factor. This directly yields a relationship between $\zeta$ and $h$

$$ \zeta = 1 - \frac{h}{\Delta} + \frac{c_s}{\gamma_s \Delta} \left(1 - e^{-\gamma_s h}\right). \quad (7) $$

The exponential approximation of the density-depth profile ($c_s$ and $\gamma_s$ in Eq. (7)) was deduced from the Vostok ice core measurements [57] and additionally confirmed in [24]. Ice properties and other model parameters used in our study are gathered in Table 2.

In terms of the above denotations, ice particle trajectories are the solutions of the following ordinary differential equations

$$ \frac{ds}{dt} = u(s, \zeta, t), \quad \frac{d\zeta}{dt} = \bar{w}(s, \zeta, t) \Delta(s, t), \quad \text{ (8)} $$

where $u$ is the longitudinal velocity and $\bar{w}$ is the apparent vertical velocity in the $(s, \zeta)$-coordinate system. The velocity field is given explicitly in [100]

$$ u = \frac{A}{\Delta} \left[1 - \sigma \left(1 - \frac{b + 2}{\beta + 1} \left(1 - (1 - \zeta)^{\beta+1}\right)\right)\right], $$

$$ \bar{w} = -b + (1 - \zeta) \left[b + w_0 + \frac{1 - (1 - \zeta)^{\beta+1}}{\beta + 1} H \frac{\partial}{\partial s} (HA \sigma)\right]. \quad (9) $$

Here $A(s, t)$ is the total ice flow rate along the reference flow line through the flow tube of a unit width

$$ A = \frac{1}{H(s)} \int_{s_0}^{s} \left(b + w_0 + \frac{\partial \Delta}{\partial t}\right) H ds $$

and $w_0(s, t)$ is the ice accretion (melting) rate at the ice-water (rock) interface. By definition, $\sigma$ is the proportion of the total ice flow rate through the flow tube due to plastic (shear) deformation of the glacier body ($0 \leq \sigma \leq 1$), and $\beta$ is the modified Glen flow law exponent, which additionally takes into account the vertical temperature gradient [61]. Three regions are distinguished along the Vostok flow line: highland area $0 < s < s_h$, the intermediate transition zone $s_h < s < s_f$ covering the vicinity of the western shoreline before the grounding line, the inlet, and the island (see Fig. 6). We assume that the $\sigma$ parameter cosine-like monotonically decreases from 1 to 0 with distance $s$ increasing from $s_h$ to $s_f$ where the ice sheet gets afloat. Both values $\beta$ and $s_h$ are considered as tuning parameters (see Table 2).

The ice sheet thickness is presented as

$$ \Delta(s, t) = \Delta_0(s) + \partial \Delta(t), \quad \text{(10)} $$

where $\Delta_0(s)$ is the present-day glacier thickness (in ice equivalent) along the flow line. Temporal ice sheet thickness fluctuations $\partial \Delta(t)$ are reconstructed after [101, 110]. This simplified multi-scale model for $\partial \Delta$ was verified, and its tuning parameters $\gamma_h$ and $\gamma_v$ given in Table 2 were constrained on the basis of the 2-D thermo-mechanically coupled model of Antarctic ice sheet dynamics [95]. Eq. (10) was later supported by 3-D simulations [96]. Another model parameter $e_h$ in Table 2 is the relative ice-mass balance excess averaged over the area of the flow tube. It is directly calculated on the basis of the spatial distribution of ice accumulation rate.

![Figure 7: The normalized ice flow tube width $H(s)$ vs. distance from Ridge B and the stacked present-day profile of the ice thickness $\Delta_0(s)$ along VFL (Fig. 4) in equivalent of pure ice with two possible versions of the intermediate part (see text).](image-url)

The boundary between the glacial and refrozen ice (accreted ice thickness $\Delta_0$) along the reference flow line over the lake is simultaneously determined from the accreted ice mass balance equation:

$$ \frac{\partial \Delta_0}{\partial t} + \frac{1}{H} \frac{\partial}{\partial s} \left(H A \frac{\partial \Delta_0}{\partial s}\right) = w_0. \quad (11) $$

Figure 7: The normalized ice flow tube width $H(s)$ vs. distance from Ridge B and the stacked present-day profile of the ice thickness $\Delta_0(s)$ along VFL (Fig. 4) in equivalent of pure ice with two possible versions of the intermediate part (see text).
along VFL are plotted in Fig. 7. Based on the wide-angle reflection technique, the ice thickness at Vostok has been slightly re-estimated as 3775±15 m in [85], being also in agreement with other observational data. This is not far (within the 20-meter uncertainty of RES measurements) from the previously deduced value of 3755 m [64, 66] which is still used here for consistency with our earlier studies and transforms to 3722 m of the ice equivalent thickness under Vostok Station in Fig. 7. The most accurate 106-km profile of the glacier bottom upstream of Vostok (solid red line) is the 5-km running average of the RES data from Fig. 6b. The rest dashed (black and red) part of the bedrock relief corresponds to the residual cartographic data from Figs. 4 and 5 smoothed in 30-km scale. The intermediate red-dotted-line fragment overlapping the dashed black profile is the detailed 10-km running average of the bedrock relief from the airborne radar transect [117, 118] along the 100-km part of the SPRU/NSF RES route [19, 98] passing closely to VFL (see Fig. 4). The red (solid, dotted and dashed) stack presented in Fig. 7 is used further as the most reliable available description of the ice sheet base relief over VFL.

4.3. Heat transfer equation

A general equation of convective heat transfer in an ice sheet in the shallow-ice approximation [31, 87] combined with the ice flow line description [86, 91] in the (s, ζ)-coordinate system takes the form [100, 107]:

\[ \rho_0 c_0 \left( \frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T \right) = \nabla \cdot \left( \lambda \nabla T \right) + Q, \]

where the ice-sheet surface slope along the flow line \( \partial l/\partial s \) over the grounded part of the glacier is [130]

\[ \left| \frac{\partial l}{\partial s} \right| = \frac{1}{g \rho_0 \Delta} \left( \frac{\nu}{\alpha} \right)^{\frac{1}{2}}, \]

and \( g \) is the gravity acceleration, \( \alpha \) is the original Glen creep exponent in the ice flow law. A scaling parameter \( \nu \) is proportional to non-linear ice viscosity. Its value given in Table 2 is fitted to the observed increase in the ice sheet surface elevation upstream from Lake Vostok to Ridge B (~290 m, see Fig. 4); the zero slope \( Q = 0 \) is assumed over the deep water area \( s_f < s < s_0 \).

4.4 Boundary conditions

To complete the ice flow and heat transfer problem (7)-(12), boundary conditions should be imposed on the free ice-sheet surface and at the glacier bottom. Originally, Eq. (12) does not account for enhanced thermal resistance of snow and firn deposits in a relatively thin near-surface stratum of the ice sheet. As discussed in [107], a special heat balance equation should be formulated at the surface in this case to preserve the heat flow and to link it with the surface temperature \( T_s \).

\[ \frac{\chi}{\Delta} \frac{\partial T}{\partial \zeta} \bigg|_{\zeta=1} = T_{s|\zeta=1} - T_s, \]

where \( \chi \) is the effective heat transfer coefficient expressed via relative density and thermal conductivity of snow and firn. Parameter \( \chi \) was estimated for Vostok area in [101] on the basis of observational data [126, 136]. Its value, given in Table 2, was later validated on the new field measurements [111] confirmed in [29].

Long-term paleoclimatic temperature variations penetrate through the ice sheet thickness into the underlying rocks, and a conjugated boundary value problem should be formulated to describe the thermal state of the glacier at the ice-rock interface [93]. Preliminary computational experiments [130] show that, for realistic estimates of the geothermal flux \( q_0 \) in Antarctica [116], the grounded part of the ice sheet upstream of Vostok is frozen to the bed \( (\omega_0 = 0) \). Consequently, it is convenient to apply the integral Laplace transform and the Duhamel theorem of operational methods [9] to the heat conduction equation in the rocks and express the heat flux at the glacier bottom as a convolution integral

\[ \frac{-\lambda}{\Delta} \frac{\partial T}{\partial \zeta} \bigg|_{\zeta=0} = q_0 - \frac{\chi}{\lambda_0 a_0} \frac{\partial}{\partial t} \int_{-t_0}^{0} \left[ T(s, \zeta = 0, \tau) \right] \frac{d\tau}{\sqrt{t-\tau}} \]

for \( 0 < s < s_1 \).

Here \( a_0, \lambda_0 \) are the thermal diffusivity and conductivity of the rocks [73, 95]. Linear vertical temperature distribution, consistent with the geothermal flux \( q_0 \) and surface temperature \( T_s \), is set as an initial temperature field at a time \( t = -t_0 \) far in the past.

Over the lake, \( s_1 < s < s_0 \), the glacier base is at the melting point,

\[ T_{s|\zeta=0} = T_f, \]

where \( T_f \) is the water freezing temperature. Given that the ice equivalent thickness under Vostok is frozen to the bed \( (\omega_0 = 0) \), the grounded part of the ice sheet upstream of Vostok is frozen to the bed \( (\omega_0 = 0) \). Consequently, it is convenient to apply the integral Laplace transform and the Duhamel theorem of operational methods [9] to the heat conduction equation in the rocks and express the heat flux at the glacier bottom as a convolution integral

\[ \frac{-\lambda}{\Delta} \frac{\partial T}{\partial \zeta} \bigg|_{\zeta=0} = q_0 - \frac{\chi}{\lambda_0 a_0} \frac{\partial}{\partial t} \int_{-t_0}^{0} \left[ T(s, \zeta = 0, \tau) \right] \frac{d\tau}{\sqrt{t-\tau}} \]

for \( 0 < s < s_1 \).

Here \( a_0, \lambda_0 \) are the thermal diffusivity and conductivity of the rocks [73, 95]. Linear vertical temperature distribution, consistent with the geothermal flux \( q_0 \) and surface temperature \( T_s \), is set as an initial temperature field at a time \( t = -t_0 \) far in the past.

Over the lake, \( s_1 < s < s_0 \), the glacier base is at the melting point,

\[ T_{s|\zeta=0} = T_f, \]

where \( T_f \) is the water freezing temperature.
with the ice fusion temperature \( T_f \) linearly related to pressure \( p \)

\[
T_f = T_{f0} - \kappa_f p .
\]

The fusion temperature at zero pressure \( T_{f0} \) and pressure depression coefficient \( \kappa_f \) depend on the lake water salinity and the concentration of dissolved air (gases) [54]. Their values resulting in \( T_f \) most consistent with the borehole temperature measurements at Vostok are given in Table 2.

The accretion ice at the contact of the ice sheet with the lake water is generally formed of the local water frozen due to the upward heat flux (cooling) though the glacier body but, in principle, we do not exclude a possibility that a certain amount of frazil ice [76, 77] can be brought by ascending fresh water flow from the colder northern ice-melting area [117, 125]:

\[
w_0 = w_f - \frac{1}{\rho_0 L_f} \frac{\lambda}{\Delta} \frac{\partial T}{\partial z}_{s=0},
\]

where \( w_f \) is the rate of frazil formation and \( L_f \) is the latent heat of ice fusion. The accretion rate \( w_0 \) is assumed to be zero over the island.

4.5 General algorithm and strategy of computations

The age \( t_d \) of an ice particle at a depth of \( h_d \) related to the \( z \) level by Eq. (7) under Vostok Station at \( s = s_0 \), that is the ice core time scale, is determined from equation \( \xi(t_d) = 1 \), where the ice particle trajectory \( (\xi(t), s(t)) \) is the backward-in-time solution of Eqs. (8)-(10) at the initial conditions \( s(0) = s_0 \), \( \xi(0) = \xi_0 \). Correspondingly, the deposition site of the particle is \( s_d = s(-t_d) \).

Implicit finite-difference schemes were used to solve the differential equations (11) and (12) at the boundary conditions (13)-(16). "Left-angle" first-order approximations along the \( s \)-axis provide stability of the computational algorithm based on the sweep method along the \( \xi \)-axis.

The climatic input of the model over the recent 420-kyr history covered by the Vostok deuterium record [78] is represented by Eqs. (2), (3), and (6) and the iteratively calculated glaciological time scale. For earlier times, the scaled geophysical metronome (5) is employed to extrapolate the inversion temperature variations in Eqs. (3) and (6) into the past.

An interactive computer system was developed to perform necessary numerical experiments in order to constrain the improved thermo-mechanical ice flow line model and its climatic input by available geophysical and glaciological data. Our previous studies [101, 112, 130, 131] show that the simulated features of the ice-flow and temperature fields in the ice sheet are selectively sensitive to different model uncertainties. Thus, the general strategy of the ice core data interpretation was in sequential and iterative fitting the model to various independent sets of observational data (borehole temperature measurements, accumulation rates deduced form air bubble properties, ice age markers, isochronous reflection layers, and other sources of glaciological information) to come to the most consistent (best-fit) model predictions and, consequently, to more reliable paleoclimatic reconstruction and ice core age dating. The following general guidelines were used in our complex optimization procedure.

1. A sufficient set of age markers statistically independent and uniformly distributed versus depth at a certain site under consideration \( s = s_0 \) reliably constrains the ice-volume flow rate \( A(s = s_0, t) \) through the prescribed reference flow tube in Eqs. (9). For a given present-day accumulation \( b_0 \) at Vostok, the flow rate determines the isotope/temperature slope \( C_T \) in the precipitation model (2) and (3) and allows tuning the shape of the spatial profile \( B(s) \) to deduce the best-fit ice age-depth relationship.

2. Although limited in distance, the isochronous reflection layers (see Fig. 6), especially in the upper, near-surface part of the ice sheet, also deliver unique information on the \( B(s) \)-distribution along the flow line. The local configuration and the total descent of isochrones in the deeper part of the glacier within the transition zone \( s_0 < s < s_3 \) at the western side of the lake are mainly controlled by the modified Glen exponent \( \beta \) and the boundary coordinate \( s_3 \).

3. The temperature gradient in the deeper part of the ice sheet above the lake remembers the thermal state of the bedrock and the geothermal flux \( q_o \) over the highland area in Eq. (14) and determines the ice accretion rate \( w_0 \). Through Eq. (11), the ice flow rate (the travel time across the lake) and the accretion rate control the lake-ice thickness \( \Delta \) which, being measured at Vostok [44], delivers the information on \( q_o \). As discussed in section 2, the borehole thermometry constrains the \( C_T \)-coefficient (the product \( C/C_T \)), the scaling factor \( \alpha_p \), and the present-day surface temperature \( T_{f0} \) in Eqs. (6) after substitution of Eq. (2). The observed temperature profile extrapolated to the ice sheet bottom provides an estimate for the fusion temperature \( T_f \) and the \( \kappa_f \)-coefficient in Eq. (15).

4. Finally, the reconstructed past temperatures on the ice sheet surface and the air bubble number density measured in the upper part of the Vostok ice core can be recalculated [55, 56, 58] into local LGM-Holocene variations of the accumulation rate to determine (and/or validate) directly the temporal slope \( C_T \) in Eqs. (2) and (3). The rest (deeper part) of the available bubble number concentration profile represents the spatial distribution of the accumulation rate \( B(s) \) upstream of Vostok.

Numerous series of hundreds of computational experiments took totally about two years of systematic iterative tests and data analyses. The inferred model parameters are presented in Tables 1 and 2, and the obtained results are discussed below, in the final section.
5. Results of model constraining and ice core data interpretation

5.1 Revised glaciological time scale

The studies of ice sheet dynamics along the Vostok ice flow line and ice age predictions reviewed in section 3 were based on simplified models and suffered from the lack of accurate geographic data. Furthermore, invariable spatial distribution of the accreted ice thickness was implicitly assumed at the glacier bottom over the lake in [112]. However, the total amount of refrozen ice is directly linked to the local velocity of the ice sheet motion and both characteristics undergo paleoclimatic fluctuations. These perturb strain rates in the glacier body floating across the lake and, as a consequence, affect the ice age-depth relationship.

Preliminary series of computational experiments performed in [130, 131] showed that inaccuracy of the model and existing uncertainties in its environmental input can lead to high local errors in ice age predictions within the estimated standard deviations of ±3.6 kyr [112]. Here we use the improved 2-D ice flow line model (7)-(16) with the general description of the velocity field transition from the grounded to floating ice sheet flow pattern, ice accretion at the glacier bottom over the lake, and the recently examined floating ice sheet flow pattern, ice accretion at the glacier bottom over the lake, and the re-examined geographic data (Figs. 4 and 5) based on the recent cartographic materials and RES profiling (Fig. 6) along VFL.

Special attention is paid to better constrain the spatial distribution and temporal variations of the ice mass balance given by Eqs. (2) and (3). Available data are gathered in Fig. 8. The present-day ice accumulation rate at Vostok is assumed to be $b_0 = 2.15 \text{ cm yr}^{-1}$ which is close to the 190-year mean value [24] and provided the best-guess time scale GTS-II for Vostok ice core [112]. The air bubble number density profile measured at Vostok confirmed [56] the best-fit isotope/inversion temperature slope $C_T = 6.1 \pm 0.3^\circ C$ deduced at $\gamma_a = 4.6$ in [130, 131] with the use of the GMTS age markers and allowed reconstruction of the accumulation rate profile $\bar{b}(s)$ over ~50 km across the lake (Fig. 8, curve 1). The latter result is in agreement with the field studies by Popov and others [81, 84]. On average, the 30-35% increase in the accumulation rate at the western side of the lake is also supported by the isotopic and stratigraphic studies in pits (personal communication by A.A. Ekaykin). These estimates give at least 30% higher present-day $b$-values at 35 and 59-km distance and a 40% increase at 96 km from Vostok (Fig. 8, squares).

The mean accumulation rate over the recent 190 years ($2.25 \pm 0.04 \text{ cm yr}^{-1}$) has been directly calculated from the depth of the Tambora eruption layer observed in the pit at the end of the VFL radar route, 106 km upstream of Vostok (Fig. 8, diamond), and practically coincides with that at Vostok. The ice mass balance enhancement factor of $1.65$ estimated in [40] for Ridge B at the location of the Dome B ice core (DB site in Figs. 4 and 5) corresponds to the triangle in Fig. 8. The normalized spatial distribution of the accumulation rate deduced in [52] form the air-borne radar observations along the flight route approximately parallel to VFL and passing 40-50 km northward is also shown as curve 2. All these data considerably reduce the $\bar{b}(s)$-profile uncertainty.

To find the best-guess environmental conditions and to tune the ice sheet flow model, we used eight RES reflection (isochronous) layers L0-L7 depicted in Fig. 6b and a selected set of the most reliable age markers. In accordance with our previous analysis [112] illustrated by Fig. 3, the less accurate DH control points were excluded from consideration. It is also clear that the 39 GMTS age-depth correlation points practically cover most of the 8 age markers from [74, 75] except for the universal $^{10}$Be peak [88, 142] observed in the Vostok ice core around the 601-meter depth and reliably dated at $41 \pm 2$ kyr. Thus, the employed set of age control points was composed of the latter beryllium event and 34 GMTS ice age markers older 50 kyr from [3].

The first series of computations were performed with the detailed 106-km RES profile of the ice-sheet base (Fig. 6b) continued by the large-scale smoothed bedrock relief (Fig. 7, black and red dashed line) from the topographic map in Fig. 5. The simple spatial distribution of the normalized accumulation rate $\bar{b}(s)$ drawn by the black dashed line in Fig. 8 in accordance with the presented observational data was found to be in agreement with the near-surface RES reflection layers L0-L2 in Fig. 6b and delivered the best-fit ice age-depth relationship. However, the standard deviation (SD) of

Figure 8: The normalized accumulation rate $\bar{b}(s)$ vs. distance from Ridge B along VFL (Fig. 4) deduced from air bubble number density [56] (curve 1) and RES data [52] (curve 2); the best-fit spatial profiles (curves 3 and 4) correspond to different versions of the bedrock relief in Fig. 7. Available field observations are shown by symbols: squares - isotopic and stratigraphic studies in pits (pers. com. by Ekaykin), diamond - the depth of the Tambora eruption layer, triangle - the estimate [40] for the DB site in Figs. 4 and 5.
Figure 9: Comparison of the best-fit simulated isochrones (red lines) dated 11.2, 23, 37.5, 73, 94.7, 126, 150, and 210 kyr with the respective reflection layers L0 to L7 (black lines) from Fig. 6b. Accreted ice layer is shaded.

the simulated deeper isochrones from the RES reflection layers L3-L7 (Fig. 6b) reached 6-8 kyr. The calculated timescale deviated from the selected age markers by 5.0-5.5 kyr, exceeding considerably the GMTS approximation level of ~4.5 kyr by GTS-II achieved in [112]. More sophisticated geographic profiles of the accumulation did not improve the convergence of the modelled ice ages to the selected control points.

Much better results were obtained when the intermediate part of the bedrock elevation profile was detailed by the airborne radar record [117, 118] along the adjacent route [20, 98] as shown in Fig. 7 (red-dotted line). Fig. 9 demonstrates a good agreement between the simulated and observed isochrones achieved in this case at age SD of 1.3, 2, 2.1, 2.5, 2.9, 2.2, 5.4, and 3.8 kyr for the layers L0 to L7, respectively. The depth deviations are 31, 20, 23, 28, 45, 46, 36, and 24 m, being comparable on average with errors of RES data interpretation [85]. Correspondingly, the best-fit model parameters $\beta = 6$ and $s_b = 310$ km were deduced.

Computational experiments show that isochrones near Vostok, within the 106-km PMGE radio-echo sounding section in Figs. 6 and 9, are not noticeably sensitive to short-scale spatial variations of the accumulation rate in the upstream VFL area within a 270-km distance from the ice divide. Accordingly, this part of the $S(s)$-profile was adjusted to reduce SD between the modelled (glaciological) time scale and the ice-age markers at Vostok. For the detailed stacked bedrock relief (Fig. 7, red line), we succeeded to reach the expected SD minimum of 4.4 kyr with the accumulation rate distribution plotted in Fig. 8 by the red solid line. The total ice flow rate $A(s = s_0, t)$ through the flow tube was validated on the present-day surface ice velocity of 2.00±0.01 m yr$^{-1}$ measured at Vostok by Wendt and others [138] and obtained in the simulations around 2.03 m yr$^{-1}$. For a given geothermal flux $q_0$, the ice flow could also be verified through the accreted ice thickness at Vostok.

The best-fit ice age-depth relationship is presented in Fig. 10a together with the used control points. This glaciological time scale, designated after [131] as GTS-III, summarizes the earlier efforts to improve the Vostok ice core age dating on the basis of ice flow line modeling. Different glaciological time scales are compared in Fig. 10b. The respective standard deviations of GTS-III from the average time scale and GTS-II developed in [112] are 2.7 and 3.0 kyr, being in full agreement with their estimated quality (2.2 and 3.6 kyr). SD between the newly simulated time scale
GTS-III and GTS-I proposed in [75] is 3.5 kyr and remains within the ±3.5-kyr deviation limits related, as discussed in [131], to uncertainties in the paleoclimatic and geographic conditions and differences in the employed thermo-mechanical models. GTS-I deviates from the 35 age markers used in our study by 5.0 kyr on average (comp. with the 4.4-kyr SD of GTS-III). At the same time, GTS-I and GTS-III have comparable statistical validity characterized by the mean variances of 2.3 and 2.8 kyr with respect to the eight age markers used in [75]. From this point of view, GTS-III based on a substantially improved modeling approach with more accurate and broader data scope may be considered as a further step towards developing a glaciological time scale for the Vostok ice core with the age errors on the order of 2 kyr or less on average down to a depth of 3310 m. The new ice age-depth relationship is presented in Table 3 together with the ice particle deposition sites given as distances from Vostok.

Necessity of the sophisticated modeling and importance of our knowledge about the environmental (paleoclimatic and geographic) conditions of the ice sheet flow for correct ice age predictions were discussed and demonstrated earlier in computational experiments [131]. Here to emphasize and illustrate this point, the preliminary best-fit time scale simulated for the roughly smoothed part of the bedrock (Fig. 7, black dashed line) and the simplified spatial distribution of the accumulation rate (Fig. 8, black dashed line) is also shown in Fig. 10b by the black dashed line. Corresponding changes in the ice age estimates around 120-170 kyr (~1600-2200 m depth interval) reach 10 kyr. Numerical tests show that this discrepancy is caused by the substantially lesser ice thickness around 170-190 and 230-250 km from the ice divide in the case of the smoothed bedrock profile. The systematic shifts between GTS-I, II and III timescales within 70-170 kyr period are also mainly due to differences in the bedrock topography and spatial distribution of the accumulation rate.

In spite of the ice flow disturbances observed in 3310-3330 m depth interval [78], recent studies of the deepest part of the Vostok glacier ice [120] showed that the glacial stages 14 and 16 are still discerned in the dust concentration record within the depth intervals 3380-3405 m and 3435-3450 m, respectively, and most likely the interglacial stage 17 covers the depth range from 3460 to 3470 m. This suggests that the ice, at least to a depth of 3470 m, has undergone only local perturbations [89]. Hence, an extension of the best-fit time scale GTS-III toward the boundary with the accreted lake ice, to a depth level of 3540 m [44], can provide a new better estimation of possible maximum variations of ice ages. The statistically expected age-depth distribution is given in Table 3 and plotted in the inset in Fig. 10a. For example, the ice age at the 3530 m depth, 10 m above the contact with accreted ice, could reach ~1000 kyr, provided that the basal ice flow had not been disturbed. The depth ranges of the stages 14, 16, and 17 suppositionally observed in the dust record [120] and their respective durations (505-550, 625-650, 665-700 kyr) estimated after [4] are marked by rectangles in the inset and indicate that the deeper part of GTS-III may underestimate ice ages by ~20-40 kyr. It should be emphasized here that the real basal ice deformation history is much more complicated than that modelled in our simulations and the time scale extension is given just to illustrate a possible tendency of the ice age growth with depth.
5.2 Paleoreconstructions

A simplified quasi-one-dimensional heat transfer model was used for temperature simulations in the vicinities of Vostok Station discussed in section 2. In particular, longitudinal heat convection was neglected in paleoclimatic borehole-temperature interpretations [3, 101, 104, 113]. However, significant geographic changes in ice accumulation rate, almost twofold increase in ice thickness (see Figs. 6-8), and considerable temperature contrast between the "cold" grounded and "warm" floating parts of the glacier along VFL result, as shown below, in large horizontal temperature gradients in the near-bottom ice stratum. The new series of computational experiments were started in [130, 131] on the basis of the 2-D flow line description of heat transfer processes in ice sheets to additionally constrain and/or validate the principal parameters $T_{0b}$, $C_T$, $C_p$, $\alpha_p$ of the climatic sub-model (2)
Following [130], paleotemperatures were deduced from the continuous temperature profile measured by R. Vostretsov in the record 3620 m deep borehole at Vostok. The preliminarily processed data [3] were shifted by 0.6°C to minimize the systematic error and to make the measurements consistent with the upper part of high-precision Rydvan's thermometry survey [106]. In accordance with [77, 122], the spatial temperature drop of 2.3°C from Ridge B to Vostok Station was introduced into Eq. (6) in $T_0$ after [131]. The present-day borehole temperature profile and mismatch between the measured and fitted temperatures are shown in Fig. 11a. SD is about ~0.03°C and compares to the accuracy of the data.

All constrained climatic parameters are given in Table 1. The present-day ice surface temperature $T_s = -58.5°C$ at Vostok is similar to that found in [101]. The deduced ice fusion temperature at the ice-sheet bottom, 3755 m below the surface, $T_f = -2.7°C$ is in agreement with [54]. At the same time, the inferred estimates of $C_i = 0.79$ (the product $C_iC_T = 4.8 \text{‰/}°\text{C}^{-1}$) and $\alpha_p = 0.06$, being close to those of [130, 131], differ considerably from the previous studies [3, 101, 104, 113] based on the simplified models and less accurate or limited-in-depth borehole temperature measurements.

Another temperature profile was measured at Vostok by C. Rado (personal communication by J.R. Petit) at the end of 1997 down to 3420 m after 8-month break in drilling operations. Being in agreement with Rydvan's measurements in its upper part, this survey reveals 0.2-0.3°C colder ice with respect to the corrected Vostretsov's data in the deeper part of the borehole. As shown in [130], the best-fit fusion temperature in this case decreases accordingly with only minimum changes in scaling factors $C_i$ and $\alpha_p$ at the same SD level. Thus, the isotope/temperature transfer function and the past accumulation rates given by Eqs. (2), (3) and (6) with the recommended parameters from Tables 1 and 2 can be considered as well established and reliably constrained.

The recovered surface and inversion temperature fluctuations are plotted in Fig. 11b. In agreement with [130, 131], the amplitudes of $\Delta T_i$ are noticeably less than before in case of the spatially one-dimensional heat transfer models [3, 101, 104, 113]. In particular, the Last Glacial Maximum (LGM) surface temperature at Vostok is found to be -67.8°C for the non-zero surface temperature spatial gradient, that is only 11.4°C lower than the Holocene-optimum temperature -56.5°C. It was 12.5-14.0°C lower in [130, 131] for the simplified ice flow description at the grounding line, while the transitions of 15-20°C were deduced previously. Special computational tests confirm that the latter difference is caused by the considerable decrease in the ratio $w/\Delta$ in Eq. (12) along VFL neglected in the simplified descriptions as well as by the use of the more reliable

![Figure 11: Paleoclimatic model constraining. (a) The borehole temperature profile at Vostok and mismatch (dots) between the measured and calculated temperatures. (b) Past inversion and surface temperature variations from Eqs. (2) and (6) at the best-fit parameters $C_T = 6.1 \text{‰/}°\text{C}^{-1}$, $C_i = 0.79$, $\alpha_p = 0.06$ (curves 1) and from Eqs. (2) and (4) at geographic estimates $C_T = 9 \text{‰/}°\text{C}^{-1}$ and $C_i = 0.67$ [41,42] (curves 2). (c) The reconstructed ice accumulation rate and ice thickness variations at Vostok.](image-url)
and complete thermometry data instead of the temperature stack [101, 113]. Earlier, the best fit between borehole temperature measurements and simulations was generally found at a higher accumulation rate \( b_0 \sim 2.4 \text{ cm yr}^{-1} \) which corresponds to a 50-year mean value at Vostok [2, 24]. This, most likely, partly counterbalanced in one-dimensional approximation a considerable increase in accumulation rate upstream towards Ridge B.

The new estimate of the LGM-Holocene temperature increase is now substantially closer to the conventional predictions (\(-9^\circ\text{C}\)) based on geographic data, although remains about 25% higher, as might be expected from well understandable difference between the spatial and temporal isotope/temperature slopes [45]. Another interesting peculiarity is that the inferred inversion/surface temperature slope \( C_i = 0.79 \) is closer to unity than its geographic analogue \( C_i = 0.67 \) in [42] and predicts more commensurable amplitudes of temporal fluctuations of inversion and surface temperatures than the difference between their spatial variations (see Fig. 11b). These results are also consistent with the independent meteorological observations [21] (see Table 1, comp. Figs. 2b and 11b).

Accumulation rate variations determined by Eqs. (2) and (3) at the best-fit value of \( C_T = 6.1 \text{ \%C}^{-1} \) with \( \gamma_m = 4.6 \) and the corresponding changes in the ice-sheet thickness at Vostok described by the simplified model [110] are presented in Fig. 11c.

Special computational experiments showed that the use of the more general equations (1) instead of Eq. (2) did not noticeably change the discussed above paleoclimatic reconstructions. The performed analysis confirms the inferred tuning parameters of the climatic sub-model (2), (3) and (6) (bold values in Table 1) as most reliable.

### 5.3 Simulated ice flow characteristics

The simulated contemporary distribution of the surface ice velocity along VFL is presented in Fig. 12a by solid line. Its spatial variation follows the changes in configuration of the ice flow tube, while the absolute values are determined by the current accumulation rates. For the 190-year averaged present-day accumulation rate \( (b_0 = 2.15 \text{ cm yr}^{-1}) \) the calculated ice velocity at Vostok in Fig. 12a is about 2.03 m yr\(^{-1}\) and approaches 2.3 m yr\(^{-1}\) for the recent 50-year mean value \( (b_0 = 2.4 \text{ cm yr}^{-1}) \). Both estimates are in perfect agreement with the direct observations 2.00\pm0.01 m yr\(^{-1}\) [138] and 3\pm0.8 m yr\(^{-1}\) [5]. The normalized spatial profile of the surface velocity remains practically (within 5%) constant in time [130]. Relative temporal variations of the surface velocity are shown in Fig. 12b. They are practically identical for different sites along the flow line and reveal two times lower velocities around 10 kyr BP after the LGM period.

Prediction of spatially non-uniform and variable-in-time ice flow rates upstream of Vostok Station (see Figs. 12a, b) is an important result of the computations which allows a more accurate estimation of the travel time needed for ice to float across the lake. The transit time varies from its minimum of about 28 kyr to maximum of 41 kyr during interglacial and glacial periods, respectively. For example, due to essentially reduced velocities in the recent glacial period, it took about 10 kyr for the present-day Vostok ice to cross the 11 km-wide embayment near the western side of the
lake and, totally, 40 kyr to approach Vostok Station. This differs considerably from preliminary predictions discussed in [117] and from the respective values of 4 and 20 kyr found in [5] under the assumption of the constant ice velocity over the lake. The above estimates characterize the motion of the surface ice. Due to the basal drag in the embayment-island area (see Fig. 6b), the bottom ice journey is even longer and takes from 38 to 57 kyr.

Ice core studies at Vostok [44] revealed 215 m of refrozen lake ice at the glacier bottom provided that the total ice sheet thickness is 3755 m [64, 66]. At zero frazil formation rates \( w_f = 0 \) in Eq. (16), this constrains the geothermal heat flux as \( q_0 \approx 0.054 \text{ W m}^{-2} \) with accuracy not worse than \( \pm 5\% \) in agreement with the general estimates of [116] for central Antarctica. As before [130], the simulated present-day basal temperatures along the major part of the grounded ice flow line (see Fig. 12a) do not reach the melting point and remain systematically below \(-4°C\). Consequently, only two factors control the ice accretion: (1) the thermal interaction of the cold glacier (overcooled with respect to the melting point) with the lake water and (2) the convective drift of the total geothermal heating from the lake floor to the northern half of the lake due to the water circulation (e.g. [117, 125]). Without direct geothermal heating from the lake, the heat losses through the ice thickness in the southern part are fully counterbalanced by the water freezing. Contemporary distribution of the accreted ice \( \Delta \alpha \) and freezing rates \( w_0 \) along the VFL are shown in Fig. 12c by solid curves 1 and 3, respectively. In accordance with these computations, about 75 meters of the accreted ice are formed over the embayment near the western side of the lake at the maximum accretion rates of \(-6\text{-}6.5 \text{ mm yr}^{-1} \). No water freezing is assumed on the island surface, and the mean accretion rates over the lake are estimated as \( 5.5 \text{ mm yr}^{-1} \). Because the ice sheet approached the lake in the past at noticeably lesser velocities than it has now at Vostok, the latter accretion rates, although essentially lower than those predicted in [5, 76, 77], were sufficient to result in 215 meters of the refrozen ice. However, it should be noted that, in accordance with [85], the ice sheet thickness at Vostok may reach 3775 m and, then, the accreted ice layer is 235-m thick. This will increase the deduced ice fusion temperature by \(-0.4°C\) and reduce the geothermal flux to \( q_0 \approx 0.05 \text{ W m}^{-2} \). Alternatively, a frazil formation rate of \( w_f \approx 0.4 \text{ mm yr}^{-1} \) or a smaller non-freezing area on the island can result in the same thickness of the refrozen ice.

Another important peculiarity of the ice sheet - subglacial lake thermodynamic interaction is that the heat flux at the glacier bottom is not noticeably influenced by paleoclimatic surface temperature variations and remains practically constant with time [130]. As a consequence, the freezing rates and the accreted ice flow rate along the flow line do not vary much. Thus, the accreted ice thickness is sensitive and inversely proportional to the ice velocity which is directly related (through the total ice flow rate \( A \)) to the

![Image](https://via.placeholder.com/150)

**Figure 13:** Present-day ice age distribution (a) and temperature field (b) in the ice sheet along VFL. Numbers at isochrones and isotherms are ages in kyr and temperatures in °C, respectively. Accreted ice layer over the lake in (a) is shaded.

6. Conclusion

This review based on the authors’ publications [101, 102, 104, 112, 130, 131] demonstrates that the joint problem of ice age dating and paleoclimatic interpretation of ice core isotopic records can not be successfully solved without involving supplementary geographical, geophysical and glaciological information. A sophisticated thermo-mechanical ice
flow line model (7)-(16) and the computer system are developed and employed as a tool to infer the ice core history at Vostok through fitting the model to various data, which include the age-depth markers, borehole temperature profile, flow-line RES survey, air bubble measurements and additional available field observations. The principal issues of the paper are the constrained isotope/temperature and accumulation transfer functions (2), (3), (6) and the best-fit glaciological time scale GTS-III for the Vostok ice core (see Tables 1 and 3) simultaneously consistent with a wide spectra of all the considered experimental materials. The substantially improved modeling approaches with the more accurate and broader informational scope are believed to provide a higher reliability of the obtained results.

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